Conflict between Goal Programming and Robust Goal Programming In Optimal Investment Portfolio Selection

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Abstract

The problem of investment portfolio selection is viewed as one of the most important issues in financial engineering. The presentation of the mean-variance model brought about a revolution in the problem of stock portfolio selection. Although the proposed model theoretically has unique characteristics, its weaknesses hinder the use of the model in practice. For this reason, many studies have been carried out until now on model performance optimization in real world issues. In this paper, a goal multi-objective optimization model with robust fuzzy multi-objective model were compared to select optimization investment portfolio on Tehran Stock Exchange, and the results were compared.

Key words: Investment portfolio, Goal programming, Robust fuzzy goal programming

INTRODUCTION

Financial portfolio is defined as a set of assets, the optimization of which consists of a financial portfolio in order to achieve desired goals of a predetermined return, by considering constraints of risks and asset allocation intended by investor. Investment managers are generally facing two serious problems about the application of quantitative techniques (return-variance model);

The first problem is that each individual or institution embark on optimizing portfolio in accordance with specific goals, which sparked investment managers’ interest in how they can take account of more criteria for quantitative process of portfolio selection as well as “risk and return”.

The second problem with investment portfolio selection is that it is not made completely clear that parameters for optimization problem like investment return on a specific asset or investment risk of that asset before problem solving and portfolio selection. Thus, one of the main characteristics of financial issues is the nature of its parameter uncertainty. If the uncertainty of the problem parameters goes unnoticed, it is possible that a slight turmoil in the actual amount of problem parameters in proportion to a given amount during problem solving causes the optimization of final solution or its reasonability to be spoiled. Therefore, if there are indefinite parameters for a problem, we need to take parameter uncertainty into account in an attempt to validate an optimal solution of a problem.

In the last few decades, modelling and solving optimization problems have received special attention by assuming the uncertainty of problem parameters, and a variety of methods have been used in relation to uncertainty of problem parameters. One of the common approaches to problem parameter uncertainty modelling is the use of robust optimization approach which has a significant performance compared to other approaches. However, it should be noted that model robustness is not an absolute concept but it is used implicitly; that is, it is not possible to offer a general model for it, but robust optimization is recognized by its application in the improvement and
generalization of classic models. Today, the link between the concept of model robustness and fuzzy concept is the key to the solution of many complex concepts.

Background of Investment Portfolio Selection
Optimization in stochastic conditions was initiated by Bill (1955), Belman (1957), Belman and Zadeh (1957), Charens and Cooper (1959), Dantzig (1955) and Tinfe (1955) in the late 1950s, and progressed immediately both in theoretical field and algorithm field. Today, by Lasting (1991) and Bickesbay (1992), Lekovitz and Mitra (1993) and Mullvey (1995) works, much progress has been made in the problem solving of stochastic programming. Many approaches to optimization under stochastic conditions were used. In this regard, we can distinguish between three main approaches namely stochastic programming, fuzzy programming, and stochastic dynamic programming (Sahinidis, 2004). Another approach which has been lately extended to confront data uncertainty is robust optimization, in that optimization is used when the worst event happens, which may result in maximum minimum objective function. This approach seeks approximately optimized solutions with high probability. In other words, with a slight disregard of objective function, we can confirm the solution obtained. However, concerning the uncertainty of objective function coefficients, with a slight neglect of the value of the optimal objective function, we seek a solution which is more likely an actual solution better than that solution. In the early 1970s, Soister (1973) came up with a linear optimization model that yielded the best rational solution for all input data, in the sense that each data can get any value from an interval. This approach tends to solutions which are more conservative. That is to say, to ensure solution robustness, we deviate a lot from the optimality of the nominal problem in this approach (Ben-Tal and Nemirovsky, 2000). Ben-Tal and Nemirovski (1998, 1999, 2000) and El-Ghaoui (1997, 1998) focused on over-conservative state and came up with performance algorithms to solve convex optimization problems under data uncertainty. However, given that robust formula were obtained, problems are conic quadratic (Ben-Tal and Nemirovski, 1999). These methods cannot be directly used for discrete optimization problems. Bertsimas and Sim (2004) proposed a different approach to conservative level control. This approach has the advantage that it leads to a linear optimization, so it is applicable to discrete optimization models.

Since optimization issues of investment portfolio consists of values such as stock price, interest rate, risk, etc., and these values are not precisely known and can only predicted, the problems are entirely discussed within the framework of robust optimization. Thus, different researchers have done research in this regard. Haung et al. (2007) developed the worst-case VaR approach and formulated related issues as semi-definite programming. To deal with robust portfolio selection in which local information is only exit time distributional function and conditional distribution of available portfolio return, Vi Chen et al. (2011) presents a new uncertain set of the worst-case value at risk (VaR) with robust optimization approach. The proposed interval uncertain set take a good account of upward and downward deviations of data. Their main idea is to present uncertain data by considering stochastic intervals. The existing models in this literature consider a definite bound for uncertain intervals, while Chen et al. (2011) suggest uncertain interval bound in stochastic fashion. Moon and Yao (2010) attempted to solve portfolio problem with mean absolute deviation (MAD) criterion by using robust optimization approach. They used Bretsimas’s robust optimization approach. Anagrazia and Alberto Zafaroni (2008) proposed conditional value-at-risk robust optimization of investment portfolio. Their aim in this model was to minimize conditional value-at-risk.

Experimental tests of this model were performed on the Italian financial market. Yongmomon and Tawiyaw (2011) presented a paper entitled “robust mean absolute deviation optimization”. In this model, a robust mean absolute deviation (RMAD) model was introduced, which results in a linear programming reducing computational complexity. In the empirical results of this paper, a variety of conditions leading to oscillation and uncertainty of data were taken into consideration. Gregory et al. (2011) solved the problem of robust portfolio optimization under different conditions and compared the cost of robustness. They modelled uncertainty in asset return in the form of non-deterministic multidimensional (rather than elliptical) sets. Gregory et al. (2011) concluded robust optimization is the best way of solving portfolio under conditions that parameter values are unknown, variant with unclear distribution. Moreover, in a situation where the distribution of parameters is precisely unknown, they recommend stochastic programming.

Seifi (2004) proposed how to apply robust optimization approach to (single-period) stock selection. He also shows how a robust model can be adjusted for investor utility function and uncertainty of stock return rate. A robust multi-period financial portfolio optimization model using conditional value-at-risk was proposed by Yazdi et al. in 2004 at the 6th Industrial Engineering Conference. In this paper, the worst-case conditional value-at-risk WCVaR in the condition that there is just partial information on probability function of uncertain parameters was studied, aiming at minimizing WCVaR with synthetic uncertainty, bound uncertainty and elliptic uncertainty for asset return distribution. Robust financial portfolio optimization model with CAPM approach was proposed by Sajjadi et al. (2010) at the 7th Conference on Industrial Engineering. In this
research, robust optimization approach was suggested for solving multi-dimensional financial portfolio selection problem. The proposed model is linear with good computational efficiency. The linear feature of this model is considered an important advantage when complex constraints such as tax are added to problem structure.

**Goal programming model for investment portfolio**

Suppose that $J = \{1, 2, ..., n\}$ is the sum of securities intended for investment, in that return rate of each of these securities is $j \in J$ time with stochastic variable $R_j$ and the mean $\mu_j = E(R_j)$, and also $X = \{x_j, j = 1, 2, ..., n\}$ are the cost of investment in (decision variables) portfolio.

Accordingly, Lee and Chaser’s (1980) programming model is as follows;

1. $\min W_1d_1 + W_2(d_2 - d_1) + W_3(d_3 - d_2) + W_4(d_4 - d_3) + W_5d_5$

   subject to:

2. $\sum_{j=1}^{n} x_j + d_i - d_i = BC$

3. $\sum_{j=1}^{n} R_j x_j + d_i - d_i = DR$

4. $\sum_{j=1}^{n} B_j x_j + d_i - d_i = B(BC)$

5. $x_j + d_{j+1} - d_{j+1} = V_j$

6. $x_j + d_{i+1} - d_{i+1} = D_j$

7. $BC + \sum_{j=1}^{n} R_j x_j + d_i - d_i = M$

Expression (2) deals with fund constraint. Expression (3) focuses on portfolio investment return rate which must be greater than $DR$ (total portfolio earning determined by investor's opinion). Expression (4) focuses on systematic risk of portfolio. If investor predicts that market situation improves in coming days, he has to bring his portfolio beta close to market beta. In this paper, it is assumed that market's future situation is in this case, so equation (4) forecasts portfolio beta on the basis of investor's opinion. Expressions (5) and (6) deals with investor constraint in each securities as well as the goal, and eventually equation (7) focuses on total fund maximization and portfolio return. 

$W_i$ through $W_i$ suggests priorities given to goals (constraints) determined by investor's opinion.

$x_j$ is decision variable indicating the amount of money invested in $j$th sheet.

$BC$: budget allocated to investment

$R_j$ desired return obtained by $R_j = \frac{P_{t+1} - P_t + D_t}{P_t}$

$P_t$: price at $t$th time

$D_t$: dividend

$DR$: total expected earnings from investment

$B_j$: beta expected for each investment portfolio share

$B$: expected systematic risk

$V_j$: maximum investment expected by investor in $j$th sheet

$D_j$: expected amount of investment in $j$th sheet

$M$: a great number

**Fuzzy-robust optimization of investment portfolio by using goal programming**

Researchers considered this type of parameter in oscillating random number in a symmetrical interval, as they had no knowledge about the distribution form of some parameters. In robust optimization models as in Bretmis and Sim’s the middle number of intervals is counted as nominal value. In cases of real problems, it is not easy for a decision maker to precisely determine the length of an interval in which this nominal value oscillates, because the length of the interval comes with ambiguity; that is to say, if decision maker considers the length of interval upward, he increases conservative level and incurs higher cost. On the contrary, if he considers the length of an interval downward, he can go up risk taking of decision making. In addition to the discussion of balance between risk taking and cost, in reality decision maker expresses the length of an interval with ambiguity at times. To solve this problem, researchers came up with an initiative in that decision maker is able to express the length of intervals in fuzzy numbers and keep a balanced risk taking.

In this model, interval bound is expressed in fuzzy, which presents us with a linear model, right-side coefficients of fuzzy linear programming. Definitely, to solve this model
we need a change from fuzzy bound to definite bound. In the context of this problem in this paper, risk parameters and return are uncertain according decision maker. Given that precise distribution of these data is not clear, data oscillation entail ambiguity over their precise value. In this paper, this happened to the length of risk parameters and asset return. If the length of an interval of these parameters is shown by $R_j^\epsilon$ and $B_j^\epsilon$, and considering what said earlier, this expression is taken as triangular fuzzy number, it is shown as $\tilde{R}_j$ and $\tilde{B}_j$. Given this, fuzzy-robust counterpart is written as follows;

\[
\begin{align*}
(8) & \quad \min W_d d_i^+ + W_d^r (d_i^+ + d_i^-) + W_2 \sum_{j=4}^{n+1} d_i^- + W_d d_i^r \\ s.t \quad & (9) \quad \sum_{j=1}^{n} x_j + d_i^- - d_i^+ = BC \\
(10) & \quad - \sum_{j=4}^{n} B_j x_j + d_i^- - d_i^+ + Z_i \Gamma_1 + \sum_{j=1}^{n} P_j = -B (BC) \\
(11) & \quad x_j + d_i^- - d_i^+ = V_j \\
(12) & \quad x_j + d_i^- - d_i^+ = D_j \\
(13) & \quad -BC - \sum_{j=4}^{n} R_j x_j + d_i^- - d_i^+ + Z_i \Gamma_1 + \sum_{j=1}^{n} P_j = -M \\
(14) & \quad Z_1 + P_j \geq \tilde{R}_j y_j \\
(15) & \quad Z_2 + P_j \geq \tilde{B}_j y_j \\
(16) & \quad -y_j \leq x_j \leq y_j \\
(17) & \quad 0^* P_j, \quad 0^* y_j, \quad 0^* Z_1, \quad 0^* Z_2
\end{align*}
\]

The above model is a linear programming model with fuzzy resources (asymmetric model) and convertible to symmetric model. Thus, the determined fuzzy-robust counterpart model of the above model is as follows;

\[
\begin{align*}
(18) & \quad \max Z = \lambda \\
\text{st} \quad & (19) \\
W_d d_i^+ + W_d^r (d_i^+ + d_i^-) + W_2 \sum_{j=4}^{n+1} d_i^- + W_d d_i^r + \lambda Z_i \leq Z_c \\
(20) & \quad \sum_{j=1}^{n} x_j + d_i^- - d_i^+ = BC \\
(21) & \quad - \sum_{j=4}^{n} R_j x_j + d_i^- - d_i^+ + Z_i \Gamma_1 + \sum_{j=1}^{n} P_j = -DR
\end{align*}
\]

In this model, $\lambda$ is the degree (amount) of constraint fulfillment

**RESULTS OF MODEL SOLUTION**

At this point, the results of solving model are offered. The model proposed in the previous part is a linear model which is can be solved easily by Lingo. As mentioned earlier, data are relating to 20 shares in month from the beginning of 1388 Farvardin (March 2009) by the end of 1392 Esfand (March 2013), and collected from Tehran Stock Exchange.

<table>
<thead>
<tr>
<th>Share</th>
<th>Amount of investment based on model</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>0.000000</td>
</tr>
<tr>
<td>X2</td>
<td>0.000000</td>
</tr>
<tr>
<td>X3</td>
<td>0.000000</td>
</tr>
<tr>
<td>X4</td>
<td>0.000000</td>
</tr>
<tr>
<td>X5</td>
<td>0.000000</td>
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<tr>
<td>X6</td>
<td>0.000000</td>
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<td>0.000000</td>
</tr>
<tr>
<td>X9</td>
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</tr>
<tr>
<td>X10</td>
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</tr>
<tr>
<td>X11</td>
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<tr>
<td>X12</td>
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<td>X13</td>
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<tr>
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<tr>
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</tr>
<tr>
<td>X18</td>
<td>0.200000</td>
</tr>
<tr>
<td>X19</td>
<td>0.000000</td>
</tr>
<tr>
<td>X20</td>
<td>0.165865</td>
</tr>
</tbody>
</table>

Real portfolio return=0.0176541
Budget rate (in million=0.8686392 rials)
Level of budget deviation=0.1313608
Having coded the above model in Lingo software, the following result was obtained:

As it is shown in the above table of the results, the goal programming model has very great deviation from budget and return close to zero. On the other hand, in the multi-objective fuzzy-robust programming model, as the cost of the robustness of return increases (in the face of various volatilities that can be associated with great volatilities of market during those days), a declining trend is followed. However, it should be noted that this model is a multi-objective model, in that various goals are met simultaneously. This is very effective in volatility of return rate as the cost of robustness increases.

**CONCLUSION**

Goal programming model entails great deviations from budget and return close to zero, but in the robust-fuzzy multi-objective programming model, as conservative level increases, the return rate of investment portfolio decreases. However, due to the fact that various goals are met simultaneously, there are dispersions at some conservative levels.

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