Print ISSN: 2321-6379
Online ISSN: 2321-595X
DOI: 10.17354/ijssI/2017/78

Original Article

Nonlinear Thermal Buckling of Annular Nano Plates on Elastic Foundation

Mostafa Sadeghian, Hosein Kheyraei

Department of Mechanical Engineering, Mashhad Branch, Islamic Azad University, Mashhad, Iran

Abstract

In this paper, nonlinear thermal buckling of annular nano plates with an orthotropic property is studied. Using Eringen's nonlocal elasticity theory, principle of virtual work, first order shear deformation plate theory (FSDT) and nonlinear Von-Karman strains, the governing equations are derived based on displacements. The differential quadrature method(DQM) is applied to discretize the derivatives equations with a non-uniform mesh point distribution(Chebyshev-Gauss-Lobatto). A unique relation is proposed which simply relates final results of thermal and mechanical buckling analysis to each other and can calculate critical buckling temperature difference and mechanical radial load at the same time. The accuracy of the present results is validated by comparing the solutions with those reported of the available reference. The effects of nonlocal parameter, radius ratio, thickness, elastic foundation are investigated on critical temperature difference for different boundary conditions. ΔT By increasing plates annularity,increases, because the plate stiffness increases. Also, local analysis overestimates values of ΔT.

Key words: Nonlinear, Thermal, Buckling, Nano, Annular

INTRODUCTION

Because experimental observations requires high-cost and complicate efforts, theoretical models such as atomistic methods have been used for identifying the properties of Nano structures [1]. The governing relations from these methods such as Eringen's nonlocal elasticity, are relatively simple and for small-scale effects in nano-scale structures have been considered. Eringen revealed that in nonlocal continuum mechanics, stress is dependent on strain in all over of continuum environment [2]. Recently, buckling analysis has attracted the attentions of scientists [3,4,5]. Among the recent similar studies, the paper of Jabbarzadeh and Sadeghian can be mentioned which they consider the buckling behavior of circular Nano plates under mechanical load on elastic foundation. In their article, results of analyses based on local and non-local theories are compared [5].

Access this article online

www.ijss-sn.com

Month of Submission : 06-2017
Month of Peer Review : 06-2017
Month of Acceptance : 07-2017
Month of Publishing : 07-2017

So, in this study symmetric thermal buckling analysis of orthotropic annular graphene sheets with non-linear strain is analyzed. The effects of small scale are considered using non-local elasticity theory.

GOVERNING EQUATIONS

Figure 1 A shows the annular plate and its model on elastic foundation. Based on the first-order shear deformation theory, the displacement field is defined as equation [4]:

$$u(r,\theta,z) = u_0(r) + z\varphi_r \tag{1}$$

$$v(r,\theta,z) = 0 \tag{2}$$

$$w(r,\theta,z) = w_0(r) \tag{3}$$

Where u, v and w are displacement components of each point at a distance z from the median plane, respectively, in the direction r, θ and z. Median plane displacement components are u_0 and w_0 which are the function of variable r and the expression φ_0 is the rotation elements about θ.

Using the assumptions of Von Karman nonlinear relationships strain- displacement, strain components base on displacement are obtained [4]:

Corresponding Author: Mostafa Sadeghian, Department of Mechanical Engineering, Mashhad Branch, Islamic Azad University, Mashhad, Iran. E-mail: msadeghian@mshdiau.ac.ir

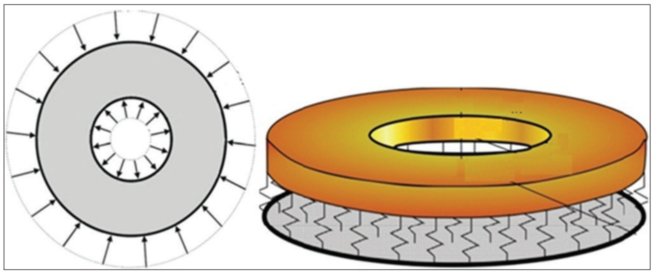

Figure 1: Annular plate and its model on elastic foundation.

$$\varepsilon_r = \frac{du_0}{dr} + z\frac{d\varphi}{dr} + \frac{1}{2}(\frac{dw_0}{dr})^2 \quad (4)$$

$$\varepsilon_\theta = \frac{u_0}{r} + z\frac{\varphi}{r} \quad (5)$$

$$\varepsilon_{rz} = \frac{1}{2}(\frac{dw_0}{dr} + \varphi) \quad (6)$$

The governing equation of nonlocal Continuum mechanics theory is presented by Eringen as follows [2]:

$$\sigma^{NL} - \mu\nabla^2\sigma^{NL} = \sigma^L \quad (7)$$

μ is nonlocal coefficient. σ^{NL} is nonlocal stress tensor and σ^L is the local stress tensor, So:

$$\begin{Bmatrix}\sigma_r^{NL}\\ \sigma_\theta^{NL}\\ \sigma_{rz}^{NL}\end{Bmatrix} - \mu\nabla^2\begin{Bmatrix}\sigma_r^{NL}\\ \sigma_\theta^{NL}\\ \sigma_{rz}^{NL}\end{Bmatrix} = \begin{bmatrix}\frac{E_1}{(1-v_{12}v_{21})} & \frac{v_{21}E_2}{(1-v_{12}v_{21})} & 0\\ \frac{v_{12}E_2}{(1-v_{12}v_{21})} & \frac{E_2}{(1-v_{12}v_{21})} & 0\\ 0 & 0 & G_{12}\end{bmatrix}\begin{Bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \varepsilon_{rz}\end{Bmatrix} \quad (8)$$

E_1 and E_2 are elasticity modulus in directions 1 and 2 and v_{12} and v_{21} are Poisson's ratio in pre-mentioned directions and G_{12} the shear modulus. The stress resultants can be defined as [2]:

$$(N_r, N_\theta, Q_r)^{NL} = \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_r{}^{NL}, \sigma_\theta{}^{NL}, \sigma_{rz}{}^{NL})dz \quad (9)$$

$$(M_r, M_\theta)^{NL} = \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_r{}^{NL}, \sigma_\theta{}^{NL})zdz \quad (10)$$

Relations between local and non-local force, moment and shear force components can be expressed as:

$$\left(1-\mu\nabla^2\right)N_i^{Nl} = N_i^l \quad (11)$$

$$\left(1-\mu\nabla^2\right)M_i^{Nl} = M_i^l \quad (12)$$

$$\left(1-\mu\nabla^2\right)Q_r^{Nl} = Q_r^l, i = (r,\theta) \quad (13)$$

N_i^l, $M_i^l, i = (r,\theta)$ and Q_r^l are the local in-plane force, moment and the shear force resultants, respectively:

$$N_r^L = \frac{E_1 h}{(1-v_{12}v_{21})}(\frac{du_0}{dr} + \frac{1}{2}(\frac{dw_0}{dr})^2) + \frac{v_{12}E_2 h}{(1-v_{12}v_{21})}(\frac{u_0}{r}) \quad (14)$$

$$N_\theta^L = \frac{v_{12}E_2 h}{(1-v_{12}v_{21})}(\frac{du_0}{dr} + \frac{1}{2}(\frac{dw_0}{dr})^2) + \frac{E_2 h}{(1-v_{12}v_{21})}(\frac{u_0}{r}) \quad (15)$$

$$Q_r^L = \frac{5}{6}(G_{12})h(\frac{dw_0}{dr}) \quad (16)$$

$$M_r^L = \frac{E_1 h^3}{12(1-v_{12}v_{21})}(\frac{d\varphi}{dr}) + \frac{v_{12}E_2 h^3}{12(1-v_{12}v_{21})}(\frac{\varphi}{r}) \quad (17)$$

$$M_\theta^L = \frac{v_{12}E_2 h^3}{12(1-v_{12}v_{21})}(\frac{d\varphi}{dr}) + \frac{E_2 h^3}{12(1-v_{12}v_{21})}(\frac{\varphi}{r}) \quad (18)$$

To determine the equilibrium equations, the principle of minimum potential energy is used:

$$\delta\Pi = \delta U + \delta\Omega \cong 0 \quad (19)$$

Where Π is the total potential energy of the system, U is strain energy and Ω is potential energy of the system of external loads. Which are defined as follow:

$$U = \frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_0^{2\pi}\int_0^r \sigma_{ij}^{NL}\varepsilon_{ij} r dr d\theta dz = \frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_0^{2\pi}\int_0^r (\sigma_r^{NL}\varepsilon_{rr} + \sigma_\theta^{NL}\varepsilon_{\theta\theta} + \sigma_{r\theta}^{NL}\varepsilon_{r\theta} + \sigma_{rz}^{NL}\varepsilon_{rz} + \sigma_{\theta z}^{NL}\varepsilon_{\theta z}) r dr d\theta dz \quad (20)$$

$$\Omega = -\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_0^{2\pi}\int_0^r N(\frac{d(ru_r)}{dr} + \frac{du_\theta}{d\theta}) dr d\theta dz \quad (21)$$

$$V_w = \frac{1}{2}\int_0^{\beta}\int_{r_i}^{r_0} K_w w_0^2 r dr d\theta \quad (22)$$

N is radial in-plane load and K_w is the Winkler coefficient of elastic foundation. The equilibrium equations in terms of the nonlocal stress resultant are obtained as follows:

$$\delta u : N_r^{NL} - r\frac{dN_r^{NL}}{dr} + N_\theta^{NL} = 0 \quad (23)$$

$$\delta\varphi : -r\frac{dM_r^{NL}}{dr} + M_\theta^{NL} + rQ_r^{NL} - M_r^{NL} = 0 \quad (24)$$

$$\delta w : Q_r^{NL} + r\frac{dQ_r^{NL}}{dr} + \frac{d}{dr}(rN_r^{NL}\frac{dw_0}{dr}) - K_w w_0 r = 0 \quad (25)$$

The equilibrium equations in terms of local stress resultants are obtained as:

$$\delta u : -N_r^L - r\frac{dN_r^L}{dr} + N_\theta^L = 0 \quad (26)$$

$$\delta\varphi : -\frac{rdM_r^L}{dr} + M_\theta^L + rQ_r^L - M_r^L = 0 \quad (27)$$

$$\delta w : Q_r^L + r\frac{dQ_r^L}{dr} + \left(1 - \mu\nabla^2\right)(-K_w w_0 r + N_r^L\frac{dw_0}{dr} + r\frac{dN_r^L}{dr}\frac{dw_0}{dr} + rN_r^L\frac{d^2w_0}{dr^2}) = 0 \quad (28)$$

In buckling analysis, neighbor equilibrium estate method is used. The equilibrium equations are obtained from small variations near equilibrium estate. The displacement, force and torque resultants are:

$$\begin{aligned} &u_0 = u_0^0 + u_0^1; w_0 = w_0^0 + w_0^1; \varphi = \varphi^0 + \varphi^1; \\ &N = N_\theta^0 + N_\theta^1; N = N_r^0 + N_r^1; Q = Q_\theta^0 + Q_\theta^1; \\ &M = M_r^0 + M_r^1; M = M_\theta^0 + M_\theta^1 \end{aligned} \quad (29)$$

0 is for the pre-buckling and 1 represents small changes in steady state. Solving pre-buckling equations:

$$N_r^0 = N_\theta^0 = -N \quad (30)$$

Furthermore the stability equations are obtained as:

$$-N_r^1 - r\frac{dN_r^1}{dr} + N_\theta^1 = 0 \quad (31)$$

$$-r\frac{dM_r^1}{dr} + M_\theta^1 + rQ_r^1 - M_r^1 = 0 \quad (32)$$

$$(1 - \propto\nabla^2)((N_r^0\frac{dw_0^1}{dr}) + (N_r^1\frac{dw_0^1}{dr}) + r\frac{dN_r^0}{dr}(\frac{dw_0^1}{dr}) + r\frac{dN_r^1}{dr}(\frac{dw_0^1}{dr}) + rN_r^0\frac{d^2w_0^1}{dr^2} + rN_r^1\frac{d^2w_0^1}{dr^2} - K_w w_0^1 r) + Q_r^1 + r\frac{dQ_r^1}{dr} = 0 \quad (33)$$

For convenience, non-dimensional expressions are defined as:

$$\begin{aligned} &u_0^* = \frac{u_0}{h}; \varphi^* = \varphi; w_0^* = \frac{w_0}{r_0}; \mu^* = \frac{\mu}{r_0^2}; \delta = \frac{h}{r_0}; R = \frac{r_{in}}{r_0}; \\ &\alpha = \frac{E_2}{E_1}; \beta = \frac{G}{E_1}; N^* = \frac{N}{E_1 h}; N_i^* = \frac{N_i}{E_1 h}; \nabla^2 = \frac{\nabla^{*2}}{r_0^2}; \\ &M^*_i = \frac{M_i}{E_1 h^2}, i = (r,\theta); Q^*_r = \frac{Q_r}{E_1 h}; K_w^* = \frac{K_w r_0}{E_1} \end{aligned} \quad (34)$$

Eventually, a unique relation is proposed which simply relates thermal and mechanical buckling analysis:

$$\Delta T = \frac{N(1 - v_{12}v_{21})}{\alpha^T(1 + \alpha v_{12})} \quad (35)$$

Where in equation (35), ΔT is critical buckling temperature difference and N is critical mechanical load.

In order to solve the nonlinear eigenvalue equation, an iterative procedure should be used for solving and the critical temperature rise values from the two subsequent iterations to satisfy convergence criteria as[6]:

$$\frac{\left|\Delta T^{r+1} - \Delta T^r\right|}{\Delta T^r} \le \varepsilon_0 \quad (36)$$

Where ε_0 is a small value and in the present analysis, it is taken to be 10^{-4} .

NUMERICAL RESULTS

To determine the numerical results, the orthotropic annular single layer, thickness h=0.335 nm, outer radius r_0=5 nm, elasticity modulus E_1=1765 *Gpa*, E_2=1588 *Gpa* also v_{12}=0.3 and $\alpha^T = 1.1 * 10^{-6}(1/K)$ are considered Poisson and thermal coefficients. In figures which does not mention directly, *R*=0.2, δ=0.1 and K_w=1(*GPa/nm*) are considered. Differential quadrature method is used [7]. Since result of numerical differential quadrature method is dependent on the number of nodes, so the convergence results of the present study is illustrated in Figure 2. The desired convergence is achieved after 9 nodes.

First to check the accuracy of the results, non-dimensional thermal parameter is defined as: $\lambda = 12(1+\nu)\Delta T\alpha^T(r_0/h)^2$. First the isotropic circular plate is considered and compared with references. Based on Table 1 the present results are in good harmony with those reported.

Variations of critical buckling temperature to nonlocal parameter for various conditions are plotted in Figure 2. As can be seen, by increasing the nonlocal parameter, ΔT

Table 1: Comparison of present results (for circular plates) of thermal buckling parameter with references for different δ

B.C	Reference	δ			
		0.001	0.01	0.05	0.1
C	Present	14.681	14.675	14.529	14.09
	[8]	14.6842	14.6842	14.6842	14.6842
	[9]	14.681	14.675	14.529	14.09
	[10]	14.681	14.674	14.501	13.988
S	Present	4.197	4.197	4.185	4.148
	[8]	4.2025	4.2025	4.2025	4.202
	[9]	4.197	4.197	4.1852	4.148
	[10]	4.197	4.197	4.1844	4.144

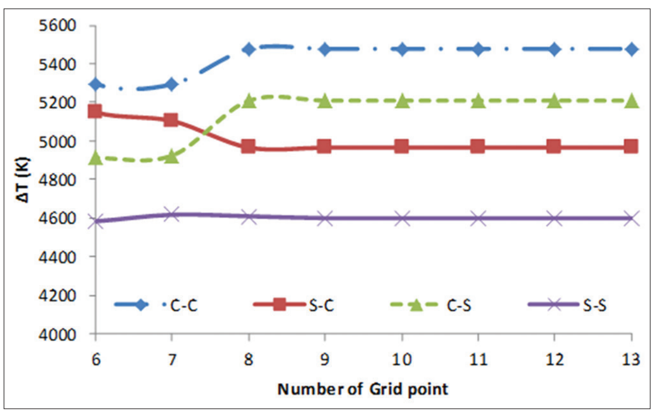

Figure 2: Convergence of ΔT for different conditions

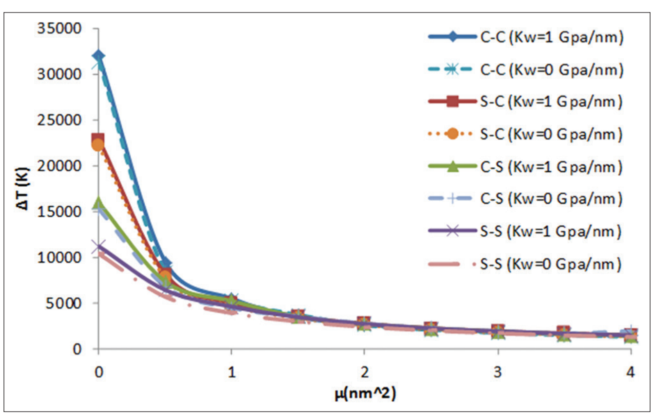

Figure 3: Changes of critical buckling temperature to nonlocal parameter for various conditions

decreases. Moreover, by increasing nonlocal parameter, the values of ΔT for different conditions approach to certain value. Also, it can be concluded by increasing the rigidity of plates (in terms of boundary condition), ΔT increases. It is also apparent that elastic foundation increases the critical buckling temperature difference of the plate.

Figures 4,5,6 and 7 illustrate changes of critical buckling temperature to nonlocal parameter for various radius ratios in C-C, S-S, S-C and C-S. As can be seen from these figures, by increasing nonlocal parameter, ΔT decreases. On the other hand, by increasing the radius ratio, ΔT increases, too. In other words, by increasing plates annularity, ΔT increases because the plate stiffness increases. While in C-C condition values of ΔT are the highest, in S-S condition are the lowest.

Changes of critical buckling temperature to thickness for various nonlocal parameters in C-C, S-S, S-C and C-S are shown in Figures 8,9,10 and 11. As can be illustrated from these graphs, by increasing nonlocal parameter, ΔT decreases. On the other hand, by increasing

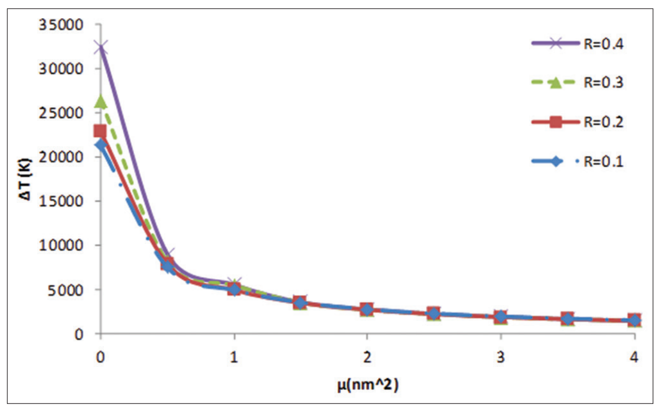

Figure 4: Changes of critical buckling temperature to nonlocal parameter for various radius ratio (C-C)

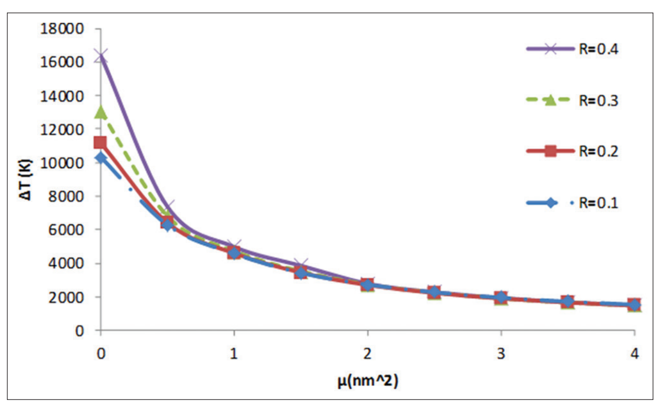

Figure 5: Changes of critical buckling temperature to nonlocal parameter for various radius ratio (S-S)

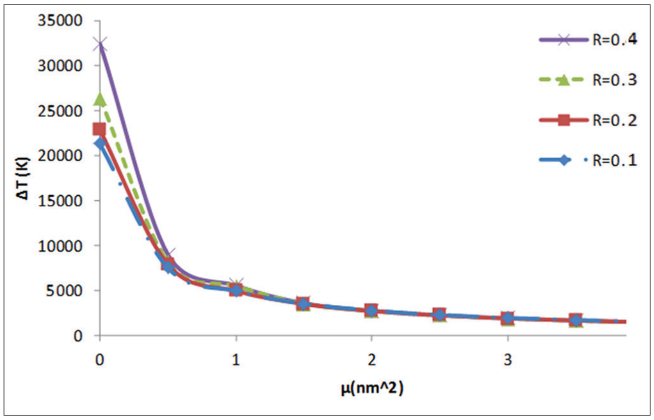

Figure 6: Changes of critical buckling temperature to nonlocal parameter for various radius ratios (S-C)

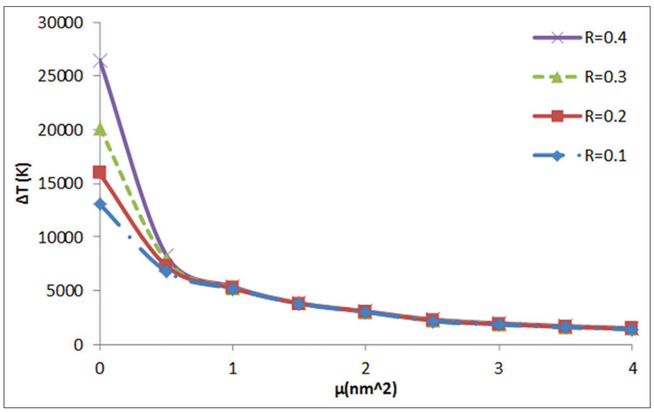

Figure 7: Changes of critical buckling temperature to nonlocal parameter for various radius ratios (C-S)

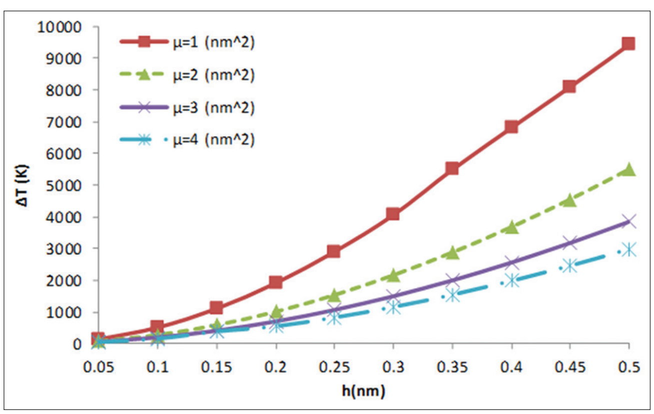

Figure 8: Changes of critical buckling temperature to thickness for various nonlocal parameters (C-C)

thickness,increases, too.values in C-C are the highest but in S-S are the lowest.

From Figure 12, it can be observed that in local state (μ=0) and for various boundary conditions are higher than nonlocal state and their values are relatively stable.

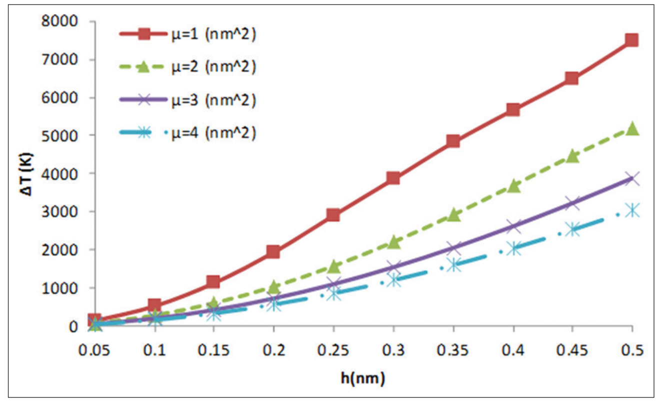

Figure 9: Changes of critical buckling temperature to thickness for various nonlocal parameters (S-S)

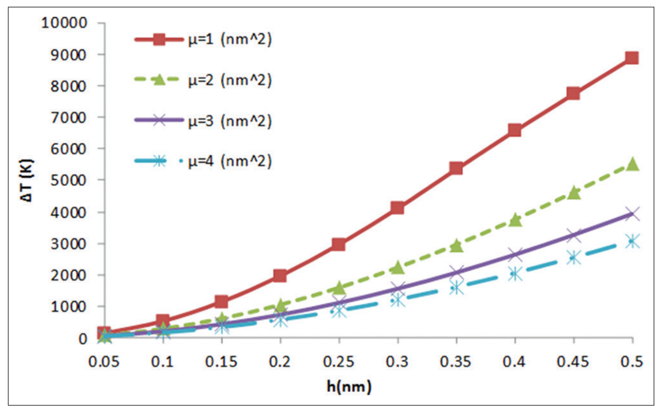

Figure 10: Changes of critical buckling temperature to thickness for various nonlocal parameters (S-C)

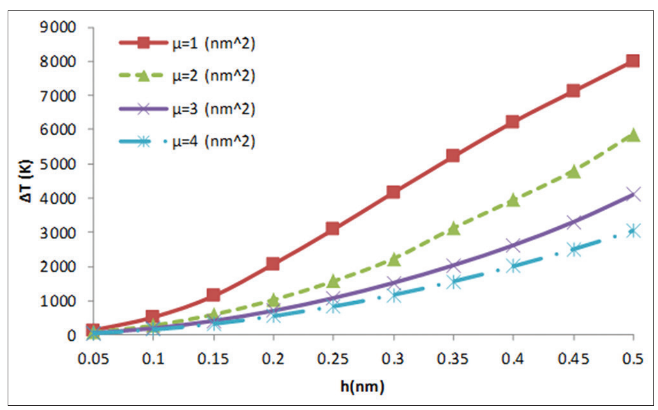

Figure 11: Changes of critical buckling temperature to thickness for various nonlocal parameters (C-S)

CONCLUSIONS

In this part, significant results of nonlinear symmetric thermal buckling analysis of annular graphene plates with nonlocal elasticity theory are mentioned as follow:

- By reduction in flexibility of boundary conditions, the effect of nonlocal parameter in the critical buckling temperature difference is more significant.

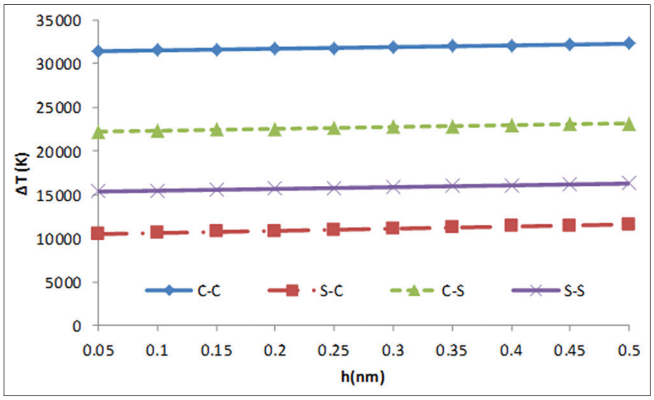

Figure 12: Changes of critical buckling temperature to thickness in $\mu=0$

- The increase of nonlocal parameter, will reduce the critical buckling temperature difference.
- Elastic foundation increases the critical buckling temperature difference of the plate.
- By increasing plates annularity, ΔT increases, because the plate stiffness increases.
- By increasing thickness, the critical buckling temperature difference, increases.
- In local analysis, ΔT is higher than nonlocal analysis. In other words, local analysis overestimates values of ΔT.

REFERENCES

1. B. Arash, Q. Wang, A review on the application of nonlocal elastic models in modeling of carbon nanotubes and graphenes, Computational Materials Science 51 (1) (2012) 303-313.
2. A. C. Eringen, Nonlocal continuum field theories, Springer, New York (2002).
3. A. A. Jandaghian, O. Rahmani, On the buckling behavior of piezoelectric nanobeams: An exact solution. Journal of Mechanical Science and Technology 29 (8) (2015) 3175-3182.
4. A. Nosier, F. Fallah, Non-linear Analysis of Functionally Graded Circular Plates under Asymmetric Transverse Loading, International journal of non-Linear mechanics, 44 (8) (2009) 928-942.
5. M. Jabbarzadeh, M. Sadeghian, Nonlinear Buckling of Circular Nano Plates on Elastic Foundation, International Journal of Engineering (IJE), TRANSACTIONS B: Applications , 29, (5), (2016) 697-705
6. A.R. Vosoughi, P. Malekzadeh, Mo.R. Banan, Ma.R.. Banan, Thermal post buckling of laminated composite skew plates with temperature-dependent properties, Thin-Walled Structures, 49 (7) (2011) 913–922.
7. C. Shu, Differential Quadrature and Its Application in Engineering, Springer (2000).
8. S. P. Timoshenko, J. M. Gere, Theory of elastic stability, Mc-Graw-Hill, New York, USA (1961).
9. S. K. Jalali, M. H. Naei., A. Poorsolhjouy, Thermal stability analysis of circular functionally graded sandwich plates of variable thickness using pseudo-spectral method, Materials & Design, 31 (10) (2010) 4755-4763,
10. M. N. Ozakca Taysi, N. Taysi, F. Kolcu, Buckling analysis and shape optimization of elastic variable thickness circular and annular plates finite element formulation, Engineering Structures, 25 (2) (2003) 181-192.

How to cite this article: Sadeghian M, Kheyraei H. Nonlinear Thermal Buckling of Annular Nano Plates on Elastic Foundation. Int J Sci Stud 2017;5(4):558-563.

Source of Support: Nil, **Conflict of Interest:** None declared.