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Original Article

Studying the Risk Analysis Performance of Fuzzy Numbers Based On L-R Deviation Degree

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Abstract

Recently, the subject of ranking fuzzy numbers based on the left and right deviation degree has attracted the attention of many researchers. Still, many ranking methods have two big problems that have been neglected. In this paper, instead of counterexamples, existing deficiencies are considered and examined through Mathematical Proofs. Applying this analysis to other researches helps to prevent common error in Ranking Index operators.

Keywords: Fuzzy numbers, Ranking fuzzy numbers, Deviation degree of fuzzy numbers.

INTRODUCTION

Because fuzzy numbers are widely used in decision theory of fuzzy numbers and data analysis, ranking fuzzy numbers is given special attention. In order to obtain more accuracy and efficiency in the results of the ranking, various methods have been proposed. The theory of center of gravity is considered as a common technique for ranking fuzzy numbers. Fuzzy sets theory was first proposed in 1965 by Professor Lotfi Asghar Zadeh. In this theory, he expressed the uncertainty caused by the ambiguity of human thought. The main advantage of this theory is the ability to provide data that are uncertain. This method is also able to use mathematical operators in the field of fuzzy data as well. The application of fuzzy sets in decision making is one of the most important applications of this theory compared to the classic sets theory. In fact, fuzzy decision theory attempts to model the ambiguity and inherent uncertainties in the preferences, goals and limitations of the decision problems. In order to obtain more accuracy and efficiency in the results of the ranking, various methods have been proposed. In this paper, instead of counterexamples, existing deficiencies are considered and examined through Mathematical Proofs. Applying our analysis by other researchers helps preventing the common error in Ranking Index operators.

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THE PROPOSED ALGORITHM FOR DETECTING INCONSISTENCIES

For ranking n fuzzy numbers of L-R, A_1, A_2, L, A_n based on the proposed functions for detecting inconsistency in results of the ranking, we propose a heuristic algorithm as follows:

Step 1: Determine the reference values of X_{min} and X_{max}

Step 2: Calculate the value of ranking index of each A_i.

Step 3: Sort fuzzy numbers in an ascending order based on values of their ranking index.

Step 4: Number the fuzzy numbers again according to their ascending relationship as $A_1^{'}, A_2^{'}, L, A_n^{'}$.

Step 5: After sorting, two fuzzy numbers of $A_i^{'}$ and $A_{i+1}^{'}$ are controlled successively (consecutive) to ensure that values of their ranking satisfy the second criterion. If they satisfied the equation, then the next pair of $A_i^{'}$ and $A_{i+1}^{'}$ will be tested, but if they didn't, $A_{i+1}^{'}$ will be temporarily removed and the pair of $A_{i+1}^{'}$ and $A_{i+2}^{'}$ will be controlled. This step continues until no pair is left without testing.

Step 6: Removed fuzzy numbers need other methods for ranking in the initial set.

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Example 1: we have two fuzzy numbers of A_1= (0,2,4) and A_2= (2,3,5) :

e_1= min (0,2) = 0 and e_2 = max (4,5) = 5 (see Figure 3)

$$H(x,y) = 6.333x^2 - 7.333y^2 - 2.833x^2y + 2.333xy^2$$
$$+7.167xy - 27x + 5y + 23.333$$

$$H(e_1,e_2) = -135$$

$x \leq 0$ and $y \geq 5$ are given. Derivatives of $H(x,y)$ are as follows:

$$\begin{cases} \dfrac{\partial H(x,y)}{\partial x} = (12.666 - 5.666y)x + 2.333y^2 + 7.167 \\ y - 27 \geq 61.76, \\ \dfrac{\partial H(x,y)}{\partial y} = (-14.666 - 4.666x)y + 2.833x^2 + 7.167x \\ +5 \leq -68.33. \end{cases}$$

If $x \leq 0$ and y ≥5, therefore $H(x, y) \leq H(e_1, e_2)$. Consequently, when applying the ranking function of $F(A)$ if $x_{min} \leq e_1$ and $x_{max} \geq e_2$, then F(A_1) < F(A_2).

Example 2: we have the set of e_1= min(0,0) = 0 and e_2 = max (3,5) = 5 :

$$H(x,y) = -\frac{8}{3}y^2 - \frac{2}{3}x^2y + \frac{5}{3}xy^2 + \frac{5}{3}$$

$$H(e_1,e_2) = -58\frac{1}{3}$$

Considering $x \leq 0$ and $y \geq 5$, partial derivatives of $H(x,y)$ are as follows:

$$\begin{cases} \dfrac{\partial H(x,y)}{\partial x} = -\dfrac{4}{3}xy + \dfrac{5}{3}y^2 \geq 25, \\ \dfrac{\partial H(x,y)}{\partial y} = -\dfrac{16}{3}y - \dfrac{2}{3}x^2 + \dfrac{10}{3} + \dfrac{5}{3} \leq -25. \end{cases}$$

Obviously, when $x \leq 0\ y \geq 5$, then $H(x,y) \leq H(e_1, e_2) < 0$. This result proves that $F(A_1) < F(A_2)$, when $x_{min} \leq e_1$ and $x_{max} \geq e_2$.

Example 3: we have A_1= (0,1,2,5), A_2= (2,3,4), e_1 = 0 and e_2 = 5

H(x,y)=6.444 x^2-3.111y_2-2.889 x^2 y+0.889xy^2+13.889xy-35.222x-13.222y+44.333

$H(e_1,e_2)=-99.556$

Considering $x \leq 0$ and $y \geq 5$, partial derivatives of $H(x,y)$ are as follows:

$$\begin{cases} \dfrac{\partial H(x,y)}{\partial x} = (12.888 - 5.778y)x + 0.889y^2 \\ +13.889y - 35.222 \geq 56.448, \\ \dfrac{\partial H(x,y)}{\partial y} = (-6.222 + 1.778x)y - 2.889x^2 \\ +13.889x - 13.222 \leq -44.332. \end{cases}$$

Table 1: The results of the ranking when applying the ranking function

Set	$G(A_1)G(A_2)$	Second criterion	Results
1	0.429	0.615	It satisfies $A_1 < A_2$
2	0.231	0.333	It satisfies $A_1 < A_2$
3	0.438	0.583	It satisfies $A_1 < A_2$
4	0.438	0.533	It satisfies $A_1 < A_2$

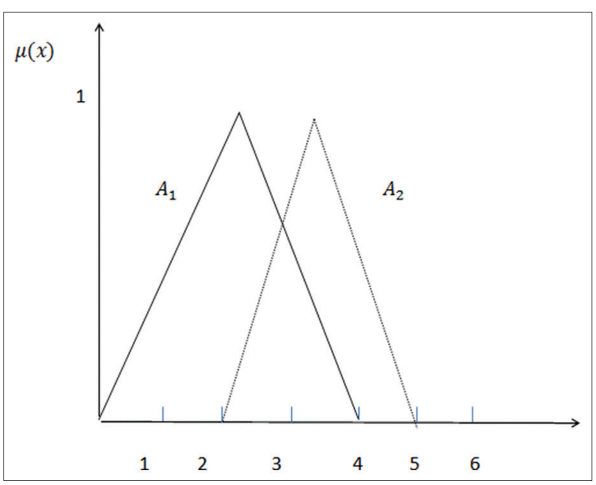


Figure 3: fuzzy numbers in example 1

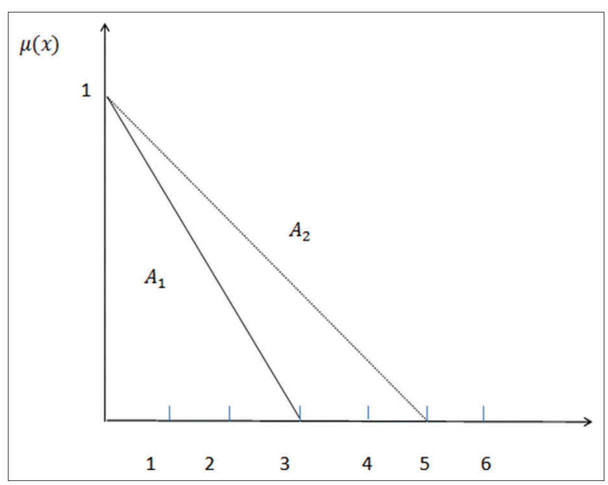


Figure (4-3): Fuzzy numbers in example 2

By following the same rules and considering $x_{min} \leq e_1$ and $x_{max} \geq e_2$, then $F(A_1) < F(A_2)$.

Example 4: we have $A_1 = (0,1,3,4)$, $A_2 = (1,2,3,5)$, $e_1 = 0$ and $e_2=5$.

$H(x,y) = 3.2x^2 - 4.2y^2 - 1.8x^2y + 1.3xy^2 + 6.65xy - 16.3x - 2.1y + 18.2$

Considering $x \leq 0$ and $y \geq 5$, partial derivatives of $H(x,y)$ are as follows:

$$\begin{cases} \dfrac{\partial H(x,y)}{\partial x} = (6.4-3.6y)x + 1.3y^2 + 6.65y \\ -16.3 \geq 49.45, \\ \dfrac{\partial H(x,y)}{\partial y} = (-8.4+2.6x)y - 1.8x^2 + 6.65x \\ -2.1 \leq -44.1. \end{cases}$$

So, $F(A_1)$, $< F(A2)$ when $x_{min} \leq e_1$ and $x_{max} \geq e_2$

To demonstrate the effectiveness (efficiency) of our algorithm, a set of fuzzy numbers are considered in the following.

Example 5: (see Figure 7): $A_1=(0,3,6)$, $A_2=(-1,0,2)$, $A_3=(0,2,4,6)$, Algorithms are used in the following format.

Step 1: $x_{max} = \max(6,2,6) = 6$ and $x_{min} = \min(0,-1,0) = -1$.

Step 2: The value of ranking index of each **A** corresponding (relevant) to the ranking functions of $F(A_i)$ and $G(A_i)$ are as follows:

$F(A_1) = 0.6197$, $F(A_2) = 0.0675$, $F(A_3) = 0.6154$.

$G(A_1)=0.55$ $G(A_2)=0.2353$, $G(A_3) = 0.4$

Step 3: Since ascending values of the ranking functions of $F(A_i)$ and $G(A_i)$ relevant to A_1, A_2 and A_3 are identical, the fuzzy numbers are sorted as A_2, A_3 and A_1.

Step 4: A_2, A_3 and A_1, respectively, are numbered again as A_1', A_2' and A_3'. Therefore $A_1' = (-1,0,2)$, $A_2' = (0,2,4,6)$,

$A_3' = (0,3,6)$.

Step 5: The pair of A_1' and A_2' have $e_1 = \min(-1,0) = -1$ and $e_2 = \max(2,6) = 6$. Using equation (30) with $P(e_1,e_2) = -29$ and $S_2^{1'} + C_1^{2'} - C_1^{1'} - S_2^{2'} = 1.5 > 0$ and $C_1^{1'} - C_1^{2'} = -4 < 0$, we can conclude that if reference values of x_{min} and x_{max} change, the values of A_1' and A_2' relevant to the ranking function of $G(A_i)$ will not be reversed.

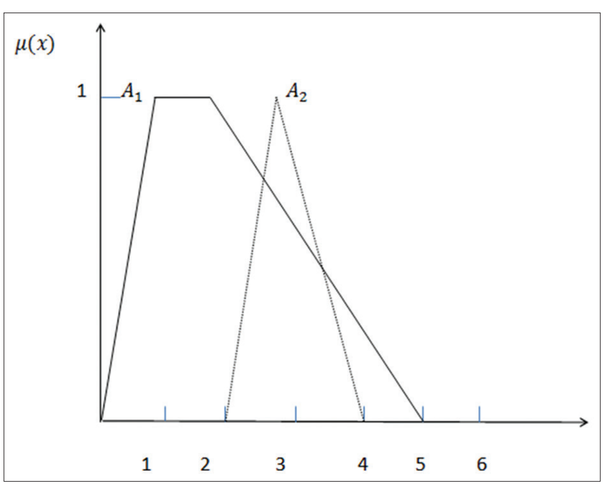
When the ranking function of $F(A_i)$ is used, we have:

$$H(x,y) = 3.17x^2 - 14.67y^2 - 4.17x^2y + 1.3xy^2 + 6.67xy - 5.33x - 3.83y + 3.5$$

$$H(e_1,e_2) = -844$$

$$\begin{cases} \dfrac{\partial H(x,y)}{\partial x} = (6.34-8.34y)x + 6.67y^2 \\ -6.67y - 5.33 \geq 238.47, \\ \dfrac{\partial H(x,y)}{\partial y} = (-29.34+13.34x)y - 4.17x^2 \\ -6.67x - 3.83 \leq -270.75. \end{cases}$$

These results prove that $x \leq -1$ and $y \geq 6$, therefore $H(x,y) \leq H(e_1,e_2) < 0$. When the reference values of



Figure(5): fuzzy numbers in example 3

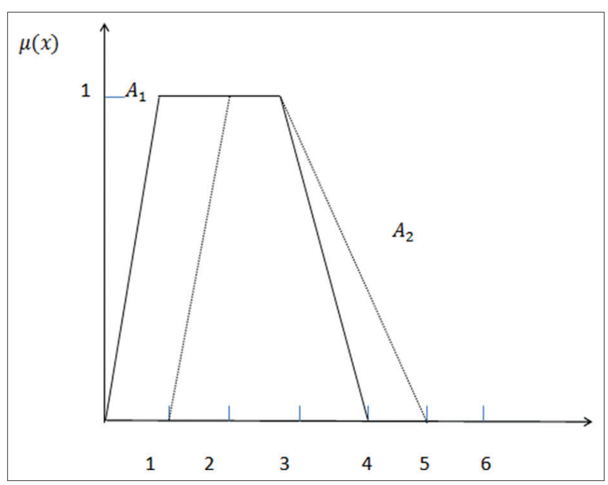


Figure (6): fuzzy numbers in example 4

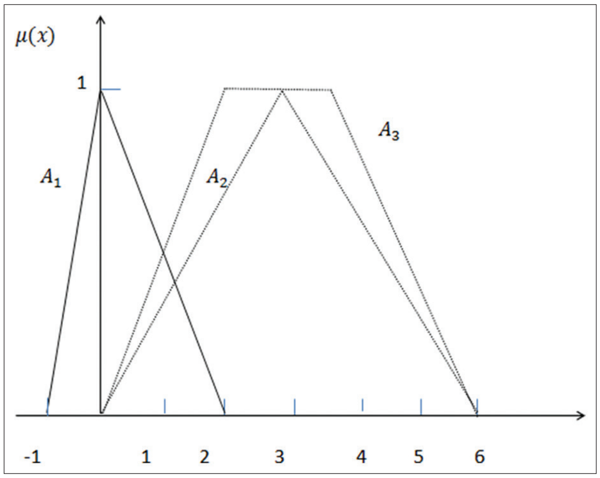


Figure 7: Fuzzy numbers in example 5

x_{min} and x_{max} change, the values of $A_1^{'}$ and $A_2^{'}$ are kept constant.

The pair of $A_2^{'}$ and $A_3^{'}$ have $e_1 = \min(-1,0) = -1$ and $e_2 = \max(6,6) = 6$. The values of $P(e_1,e_2)$ and $S_2^{2^{'}} + C_1^{3^{'}} - C_1^{2^{'}} - S_2^{3^{'}}$ and $C_1^{2^{'}} - C_1^{3^{'}}$ are, respectively, 0 and 0.5 and 0.5. since these values don't satisfy the equations (30-3) and (32-3), when the ranking function of $F(A_i)$ is used, the ranking function of $G(A_i)$ is not appropriate for ranking $A_2^{'}$ and $A_3^{'}$.

$$H(x,y) = 1.5x^2 - 1.5y^2 - 0.5x^2y + 0.5xy^2 - 6xy$$
$$-13.5x - 13.5y + 27$$

$$H(e_1,e_2) = 0$$

$$\begin{cases} \dfrac{\partial H(x,y)}{\partial x} = (3-y)x - 0.5y^2 + 6y - 13.5, \\ \dfrac{\partial H(x,y)}{\partial y} = (3-x)y - 0.5x^2 + 6x - 13.5. \end{cases}$$

Through this analysis, it is clear that the ranking function of $F(A_i)$ is unable to properly determine $A_2^{'}$ and $A_3^{'}$. For example, if only two fuzzy numbers of $A_2^{'}$ and $A_3^{'}$ will be ranked, then $F(A_2^{'}) = F(A_3^{'}) = 0.5$. If $A_1^{'}$ will also be considered, then $F(A_2') = 0.6154 < F(A_3') = 0.6197$

Now, we remove A_2' temporarily and we test the pair of A_1' and A_3'. The pair of A_1' and A_3' have $e_1 = \min(-1,0) = -1$ and $e_2 = \max(2,6) = 6$ and $P(e_1,e_2) = -26.75$ and $S_2^{1^{'}} + C_1^{3^{'}} - C_1^{1^{'}} - S_2^{3^{'}} = 2 > 0$ and $C_1^{1^{'}} - C_1^{3^{'}} = -3.5 < 0$. Since $A_1^{'}$ and $A_3^{'}$ satisfy the equation (30), $A_1^{'} \prec A_3^{'}$, while using the ranking function of $F(A_i)$, we have:

$$H(x,y) = 4.67x^2 - 13.17y^2 - 4.67x^2y + 6.17xy^2$$
$$+7.25xy - 7.25x - 3.75y + 3.75$$

$$H(e_1,e_2) = -774.33$$

$$\begin{cases} \dfrac{\partial H(x,y)}{\partial x} = (9.34 - 9.34y)x + 6.17y^2 \\ -7.25y - 7.25 \geq 305.07, \\ \dfrac{\partial H(x,y)}{\partial y} = (-26.34 + 13.34x)y - 4.67x^2 + 7.25x \\ -3.75 \leq -239.25. \end{cases}$$

So, when the reference values of x_{min} and x_{max} change, the values of $A_1^{'}$ and $A_3^{'}$ are kept constant. From the analysis of $A_1^{'}$ and $A_2^{'}$ and $A_3^{'}$, we conclude that: $A_2 \prec A_1$ and $A_2 \prec A_3$ and A_1 and A_3 can't be ranked using the ranking functions of $F(A_i)$ and $G(A_i)$.
Step 6: Ranking A_1 and A_3 in the initial set requires other methods.

CONCLUSION

In this paper, generally, two criteria for ranking fuzzy numbers of L-R were presented using the same deviation degree and similar reasoning. Using this criterion, with a detailed analysis, two ranking functions were fully presentedin order for other researchers to have a better insight for detecting the inconsistencies in their ranking functions using these criteria and our analysis.

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