

# Theoretical Analysis of the Relationship between Required Reserved Ratio and Inflation: Differential Equations Approach

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## Abstract

The present research analyzes the inflation-unemployment equations as well as the effects of required reserve ratio on money growth and inflation. Such that, initially the relationship between each of the variables is determined and then according to a new equation, the effect of the equation on inflation-unemployment equations is applied by solving differential equation. The results show that there is a negative relation between the required reserve ratio for banks with inflation rate and money growth and the type of inflation and its path is seen by replacing these relations in the equations of inflation-unemployment and the last result of this issue is that the banking based on the principle of the fractional reserve leads to severe fluctuations in inflation, which may be irreparable.

**Key words:** Required reserve ratio, Inflation rate, Differential equations

## INTRODUCTION

The history of civilization is the history of money. What can be seen in human history, is that money has had a unique role in economic developments process and caused many problems, too (Alexander Dumas).

Money is a social event that influences the economic activities of people. Today, dominant monetary system has major flows and it is immoral in many ways. Today, science within the theories framework even capitalism system has confessed to the inefficiency and instability of the monetary system and in different ways, many miserable events such as inflation, unemployment, unequal distribution of income and wealth, environmental degradation and etc. Have been made by the ruling monetary system.

In a word, enormous scientific achievements in this regard should be presented and it should be indicated that the

most common human misery is due to ignorance towards economic activities which today, the main cause is a common conception about money and common banking. Development of a so-called valid organization and being borrowed the community and paying usury in the form of beautiful words such as interest, profit, lending rate and all have made irreparable damages.

In this paper, each of new mathematical relations is presented and analyzed by using the relations between required reserve ratio, discount rate, saving rate and primary deposit and national income and etc. and by having some information from inflation-unemployment equations or Philips curve based on phillips-freidman issue.

## Research Literature

When the crisis occurred in 1930s, many economists believed that crisis causes in monetary- credit system were weak and unreliable and it was because of banking system performance based on fractional reserve principle.

To exit crisis, they recommended that the financial system should be reformed and community should be financially safe.

The founders of this domain were Frank Night (1927 and 1933), Henry Simmons (1948), Jakob Winer (1936), and Liod Mintz (1945). Then these people were known

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as founders of Chicago school and their plan was called “Chicago plan” to reform financial system in the United States.

James Toobin(1989 and 1987), Litan(1987), Spang(1989) and Berenham defended “limit banking” to improve the security of payment system and remove the costs related to current system of central deposit reserve.

Allais amendment (1984) requires that bank deposit should be 100%, according to the basic money deposits and it also should be prohibited for lending. Lending institutions based on some principles are managed which the borrowing and lending period of banks are like each other. Today, while banks borrow for short-time and give loan for long-time and, Allias requires that banks should borrow for long-term and lend for short-term.

Toobin(1985) and Ele(1984) presented that the fractional reserve banking development is largely a historical event which for creating an intermediate in the proper circulation has been out of public will. In fact, profit motivation for bankers and government need to finance have led to appearance of current banks.

Soodi(1933) represents this idea in his book that a bank generates poverty and spread it out. He puts banking against technology production and concludes that the technology makes happiness and welfare, but banking structure prevents their expansion. In his theory, there is a difference between real wealth made by raw material and virtual wealth made by bank’s promises on real wealth. To solve this financial problem, he recommended that the required reserve ratio should be 100 percent. Night emphasizes that soodi’s words in terms of economic theory is absolutely right. When money is increased by a public institution and with no charge, money is increased by commercial bank system and it is terrifying through interest payment. Night is fully agree with soodi concerning this issue that the fractional reserve banking system increases the level of prices and creates a potentially unstable situation.

### **Problem Statement**

Inflation is a continuous as well as epidemic rise process in the price if it is related to the majority of goods when it is not dramatic and temporary but continuing.

According to law, the banks are responsible for keeping a small percent of the people’s deposit as the required reserve in the central bank and they do their banking affairs and expand the volume of money with the rest of the deposits. The higher required reserve rate, the less expanding the volume of money hence the less rate leads to the more

money creation by the banks. Therefore, the central bank is able to change the required reserve by changing the increasing monetary ratio hence the volume of money.

In general, the standard banking system works based on the principle offractional reserve. In this system, in each money creation process, a percent of the given deposit to the bank is used to make the next loan. According to the economy theory, the higher proportion of this leads to the more limited possibility of money supply.

If the bank is authorized to create money, the optimum amount of money will be undetermined. Since the optimum money amount is occurred when the existing liquidity ratio in the society can be collected as necessary. Therefore, the required reserve ratios should be one hundred percent meaning the bank should not give a loan. In a situation that a bank is existed, adjusting money supply cannot be done and hence the monetary police content losses its validity very much.

In a case of a bank existence, the aforementioned money will be returned to the loaner (bank) before it is effective and provides services or goods and it is the base of another loan.

In economy, the higher number of exchanges, the higher fund transference among the accounts so the bank is more authorized to create more money hence inflation will potentially be higher.

The effect of using this tools is immediately shown in the all country banks. It should be noted that its effect is valid when the banks have no extra savings since if the value rate rises, the banks compensate the increased required reserve through their extra savings. In its falling, there was no significant expansion in the volume of money although the extra savings exist.

What can be stated is that on the first step of the money given to a bank branch as a deposit, some loan and the new deposits are created. Then, the other deposits are not increased but the loan repay rate is rose. The required reserve proportion to the loan is getting decreased by increasing the loan rate and remaining the deposits constant (or at least, a sharper increase in giving loan proportional to creating deposits) but the required reserve proportion to the deposit rates is not changed. When a new price is introduced to a bank as a deposit (producing paper currency), this reproduced paper currency leads to create more deposits hence more loans till the deposit creation processes stop while making loan processes continue.

According to this case, new deposits are not created any more but the new loans are paid; so it results in the following problems:

- Making loan through repaying the previous loans.
- Making loan through financial provision of the exchanges.

The bank loan makes the demands increase significantly and causes the exchange rate increase hence rises the unreal national income level. This issue is not necessarily meant the real national income increase in providing goods and services. The main reason for the unreal income increase lies in increasing number of exchanges based on time unit because of demand increase (for the financial provision through loan). Banking system based on the main minimum saving mechanism influences the increasing national income ratio.

First of all, the factors used this model are introduced:

- r: Required reserve rate
- s: Saving rate
- D: Primary deposit
- Y: National income
- C: National consumption

If a primary deposit, D is kept in the bank, the bank at first deducts  $rD$  from then it gives loan from the rest,  $(1-r)D$ . In economy, this loan increases the expenses proportionally. As this rate, the national income is increased too. The people after  $s(1-r)D$  deduction from, as a saving, spend the rest  $(1-s)(1-r)D$ .

To simplify the problem,  $(1-r)D$  is shown by A. The deposited price  $sA$  creates a new deposit accordingly so that the bank can give loan from the rest,  $(1-r)sA$  after deducting  $r$  percent from this loan increases the expenses proportionally. This point should be considered that the expense increases, on the second step, result from two factors:

- The first factor: increasing income because of spending the first loan  $(1-s)A$ .
- The second factor: increasing income because of spending the second loan (resulted from saving)  $(1-r)sA$ .

Therefore, the all changed expenses because of giving loan from the savings are as follows:

$$1) \quad (1-s)A + (1-r)s = A(1-s-rs) = A(1-rs)$$

If this process is continued, the third increase in expenses is occurred, as follows:

$$2) = (1-s)s(1-r)sA + (1-r)s(1-s)A + (1-s)(1-r)sA + (1-s)(1-s)A$$

$$3) = s^2 + (1-s)(1-r)s + (1-s)(1-r)s(1-rs) = ((1-r)s)^2 A$$

$$4) = ((1-r)^2 s^2 + 2(1-s)(1-r)s + (1-s)^2) A$$

$$5) = ((s^2 - 2rs^2 + r^2 s^2) + 2s(1-r-s+rs) + (1-2s+s^2)) A$$

$$6) = (s^2 - 2rs^2 + r^2 s^2 + 2s - 2rs - 2s^2 + 2rs^2 + 1 - 2s + s^2) A$$

$$7) = (1 - 2rs + r^2 s^2) A$$

$$8) = (1-rs)^2 A$$

Hence, we can express the increasing sequences of spending to the third one in this way:

The first sequence: A

The second sequence:  $(1-rs)A$

The third sequence:  $(1-rs)^2 A$

The fourth sequence:  $(1-rs)^3 A$

The nth sequence:  $(1-rs)^{n-1} A$

If  $n$  tends to  $\infty$ ,  $(1-rs)$  fraction tends to zero. Our aim is obtaining the whole national income change because of the primary deposit. So, we should sum up all the above sequences:

$$9) Y = A + A(1-rs) + A(1-rs)^2 + A(1-rs)^3 + \dots$$

$$10) Y = A[1 + (1-rs) + (1-rs)^2 + (1-rs)^3 + \dots]$$

The expression inside the bracket is an indefinite series which sum can be expressed by the geometric progression, as follows:

$$11) Y = A[1/rs]$$

Since  $rs$  is smaller than 1 ( $rs < 1$ ), the inverse value will be greater than 1 ( $> 1$ ). This ratio is the balanced increasing ratio of the national income.

### Necessity of Doing this Research

Since inflation is a long-term economic sickness and causes unoptimum designation of resources, declining the purchase power, economic inefficiency and production decrease also it brings about political, cultural and social conditions, many politicians and the economical agents pay attention to this issue. By emerging some intense inflation pressures, the price control is considered as the main aim of the economical policies by the economists. Illuminating some influence factors making these policies more efficient can pave the way to achieve the economical goals such as

inflation control, monetary policy increase by the central bank, etc.

**The Basic Objectives of Research**

In this study, we examine how the required reserve ratio influences money growth in a new approach and its effect on unemployment-inflation trend in a space of mathematical economics and by using certain differential equations that is very important in the real business cycle of school.

**Date Analysis**

Data analysis is based on solving differential equations or it is differential.

**Research Methodology**

The method of this research is a theoretical analysis. Therefore, a standard solution has been studied by some mathematicians such as Chiang, Pemberton, Rao and Todorava. The main relation of Philips shows that there are a negative relation between inflation rate and unemployment level and a positive relation with the expected inflation rate, so that:

$$12) \dot{P} = \alpha - \beta U + h\lambda \quad \alpha, \beta > 0, 0 < h < 1$$

$\dot{P} = \dot{P}/p$  is the growth rate of the price level or the inflation rate,  $u$  is unemployment rate and  $\pi$  is the expected inflation rate. When people and companies expect greater inflation, this expectation stimulates the inflation. Actually, when people expect the prices increase, they decide to buy more, right now, as people expect reduction (as an appropriate government policy), in fact, this brings a real inflation. This version from Phillips relation is called a completed expectation accounted the inflation rate. Adaptive expectations hypothesis displays how inflation expectations are formed.

The following equation:

$$13) d\pi/dt = j(\dot{P} - \pi) \quad 0 < j < 1$$

Represents when the real inflation rate is greater than expectation, this raises people's expectations  $d\pi/dt > 0$ , conversely, while the real inflation is less than (smaller) expectation, people believe that the inflation rate comes down and therefore, it reduces. If anticipation and real inflation are equal, people don't make any change in the expected inflation level.

There is also a reverse effect from inflation to unemployment. When inflation increases for a long time, people might not want to save money, therefore, capital accumulation is reduced and unemployment is increased. We can write:

$$14) dU/dt = -k(\dot{m} - \dot{P}) \quad k > 0$$

Or wherever  $\dot{m}$  is the rate of nominal money growth, unemployment rises according to real money.  $(\dot{m} - \dot{P})$  term, shows the rate of real money growth or it presents the difference between the rate of nominal money growth and the inflation rate.

$$15) \dot{m} = \dot{P} - kU$$

$$16) \dot{m} - \dot{P} = \dot{m}'/m - \dot{p}'/p = r$$

Which the real money is a nominal money divided by the average level of prices in economy.

We know that the required reverse rate increases in banks, the number of transactions decrease and the circulation of money is less. Therefore, national income and inflation are reduced, too and there is a clear inverse relation between the required reverse ratio with money growth and inflation. Therefore the equation is:

$$17) dR/dt = -L(\dot{m} + \dot{P}) \quad 0 < L < 1$$

$$18) \dot{m} = \dot{P} + \Theta R_d$$

According to aforementioned relations, unemployment-inflation relation is changed by the following equation:

$$19) \dot{P} = \alpha - \beta U + h\pi - \Theta R_d$$

Now, by replacing (19) in (20) we have:

$$20) d\pi/dt = j(\alpha - \beta U - \Theta R_d) + j(h-1)\pi$$

According to time(t) with dedifferentiation, we have:

$$21) d^2\pi/dt^2 = -j\beta du/dt - j\Theta dR_d/dt + j(h-1)d\pi/dt$$

$$22) d^2\pi/dt^2 = j\beta k(\dot{m} - \dot{P}) + j\Theta L(\dot{m} + \dot{P}) + j(h-1)d\pi/dt$$

Which based on equation (13), we have:

$$23) \dot{P} = 1/j d\pi/dt + \pi$$

And by replacing (23) in (22), we get a quadratic equation for  $\pi$ :

$$24) d^2\pi/dt^2 - [-\beta k + \Theta L + j(h-1)]d\pi/dt - [j\Theta L - j\beta k]\pi = j\beta k\dot{m} + j\Theta L\dot{m}$$

Which according to formulas, the following equation is computed:

$$25) Y'' - \alpha_1 Y' - \alpha_2 Y = \beta$$

$$26) \lambda^2 - \alpha_1 \lambda - \alpha_2 = 0$$

$$27) \lambda_1, \lambda_2 = \frac{\alpha_1 \pm \sqrt{\alpha_1^2 - 4\alpha_2}}{2}$$

We calculate the amount of radical that is shortly:

$$28) \alpha_1^2 + 4\alpha_2 = \beta^2 k^2 + \Theta^2 L^2 + j^2 h^2 + j^2 + 2\beta k(-\Theta L - jh - j) + 2j(-hj + \Theta Lh + \Theta L)$$

Model analysis:

Now, by representing assumptions, each of the statements are described:

The first statement:

$$\begin{cases} l = j = 1 \\ k = 0 \end{cases}$$

$$29) \alpha_1^2 + 4\alpha_2 = (\Theta + h)^2 + 1 - 2h + 2\Theta > 0$$

Therefore, the equation has real and distinct roots:

$$30) \lambda_1, \lambda_2 = \frac{(\Theta + h - 1) \pm \sqrt{(\Theta + h - 1)^2 + 4\Theta}}{2}$$

If  $x = (\Theta + h - 1)$ , the general answer is:

$$31) \frac{x + \sqrt{x^2 + 4\Theta}}{c_1 e^{\frac{x + \sqrt{x^2 + 4\Theta}}{2} t}} + c_2 e^{\frac{x - \sqrt{x^2 + 4\Theta}}{2} t}$$

And the specific answer is:

$$32) \alpha_{2 \neq 0}, Y_p = k, Y'_p = Y''_p = 0$$

$$Y_p = \frac{-\beta}{\alpha_2} = \frac{-\theta m}{\theta} = -m$$

According to aforementioned equations, we conclude that when  $p = 1/1+r$  is 1, it means that the interest rate should be very low and therefore, money creation is zero and inflation is reduced and finally according to money growth, the prices level is decreased.

This statement is adaptable with the intended assumptions.

The second statement:

$$\begin{cases} k = l = 1 \\ j = 0 \end{cases}$$

$$33) \alpha_1^2 + 4\alpha_2 = (\beta - \Theta) > 0$$

Therefore, the equation has real and distinct roots.

$$\lambda_1, \lambda_2 = 0, \Theta - \beta$$

the specific answers are:

$$34) Y = c_1 e^{0t} + c_2 e^{(\Theta - \beta)t}$$

Which the first part of the relation is a constant number and it means that the inflation path is constant and the second part depends on this issue that if  $\Theta - \beta$  is greater than zero, inflation is strictly increasing and if it is less than zero, inflation is strictly decreasing.

And the specific answers are:

$$35) \alpha_{2 \neq 0}, Y_p = k, Y'_p = Y''_p = 0$$

$$Y_p = \frac{-\beta}{\alpha_2} = 0$$

Since we assumed the inflation rate coefficient is zero, the results gained from solving equations are adaptable and the inflation path is constant.

The third statement:

$$36) \begin{cases} j = k = 1 \\ l = 0 \end{cases}$$

$$\alpha_1^2 + 4\alpha_2 = (\beta + h - 1)^2 - 4\beta h > 0$$

Therefore, the equation has real and distinct roots.

$$37) \lambda_1, \lambda_2 = \frac{(-\beta + h - 1) \pm \sqrt{(-\beta + h - 1)^2 - 4\Theta}}{2}$$

The general answers are as follows:

$$38) Y = c_1 e^{\frac{x + \sqrt{x^2 + 4\beta}}{2} t} + c_2 e^{\frac{x - \sqrt{x^2 + 4\beta}}{2} t}$$

And the specific answers will be:

$$39) \alpha_{2 \neq 0}, Y_p = k, Y'_p = Y''_p = 0$$

$$Y_p = \frac{-\beta}{\alpha_2} = \frac{-\beta m}{-\beta} = +m$$

The present statement is completely against the first statement. It means that the discount rate is zero, when  $p = 1/1+r$  and it means that the interest rate should be very high and therefore, very high interest rate leads to very high inflation in economy. It means that, in economy according to our equations, the inflation increases exactly equal to the money growth rate and whatever the volume of money is raised, the prices level is increased, too. In fact, this kind of inflation is because of being high the bank profits. This statement is adaptable with assumed assumptions.

The fourth statement:

$$40) \begin{cases} j = l = 1 \\ k = 1 \end{cases}$$

$$\alpha_1^2 + 4\alpha_2 = \beta^2 > 0$$

Therefore, the equation has the real and distinct roots.

$$\lambda_1, \lambda_2 = 0, -\beta$$

the general answers will be:

$$41) Y = c_1 e^{0t} + c_2 e^{-\beta t}$$

Which the first part of it is a constant number and it means that the inflation path is constant and the second part shows that the exponential function is strictly decreasing.

The specific answer is:

$$42) \alpha_2 \neq 0, Y_p = k, Y'_p = Y''_p = 0$$

Since we assumed the inflation rate coefficient is zero, the results gained from solving equations are adaptable and the inflation path is constant.

The fifth statement:

$$43) \begin{cases} k = j = 0 \\ l = 1 \end{cases}$$

$$\alpha_1^2 + 4\alpha_2 = \Theta^2 > 0$$

Therefore, the equation has the real and distinct roots.

$$\lambda_1, \lambda_2 = 0, \Theta$$

the general answers will be as following:

$$44) Y = c_1 e^{0t} + c_2 e^{\Theta t}$$

Which the first part of the relation is a constant number and it means that the inflation path is constant and the second part is an exponential function which is strictly increasing.

And the specific answer is the following relation:

$$45) \alpha_2 \neq 0, Y_p = k, Y'_p = Y''_p = 0$$

$$Y_p = \frac{-\beta}{\alpha_2} = \frac{0}{0} = 0$$

Since we assumed the inflation rate coefficient is zero, the results gained from solving equations are adaptable with assumptions and the inflation path is constant.

The sixth statement:

$$46) \begin{cases} l = k = 0 \\ j = 1 \end{cases}$$

$$\alpha_1^2 + 4\alpha_2 = (h-1)^2 > 0$$

Therefore, the equation has the real and distinct roots.

$$\lambda_1, \lambda_2 = 0, (h-1)$$

the general answers will be as following:

$$47) Y = c_1 e^{0t} + c_2 e^{(h-1)t}$$

Which the first part of the relation is a constant number and it means that the inflation path is constant and the second part is an exponential function which is strictly increasing which its scope and range start from +1.

The specific answers will be as following:

$$48) \alpha_2 \neq 0, Y_p = k, Y'_p = Y''_p = 0$$

$$Y_p = \frac{-\beta}{\alpha_2} = \frac{0}{0} = 0$$

Since we assumed the growth rate coefficient of money volume is zero, the results gained from solving equations are adaptable with the assumptions and the inflation path is constant.

## CONCLUSION

In this research, firstly in a mathematical economy space and according to differential equations, the role of banking system based on the required reserve ratio in credit creation was discussed. In general, inflation assumed in this model originates from two regions: Inflation caused by interest rate and inflation caused by money creation. According to each of the statements, the specific statements were assumed. So that, if inflation caused by money creation wiped out, we would have inflation caused by interest rate. Therefore, this kind of banking system based on the fractional reserve principle has made a basic inconstancy and also made the contraction and expansion of the national money. So that, the bank system through money creation can easily take a significant part of national creation.

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